Derandomizing Polynomial Identity Testing for Multilinear Constant-Read Formulae

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Arithmetic Formula Identity Testing

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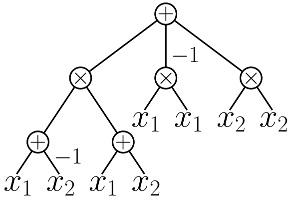

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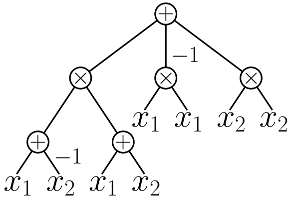

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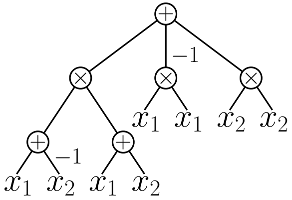

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Motivation: primality testing, circuit lower bounds, ...

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Theorem (Main)

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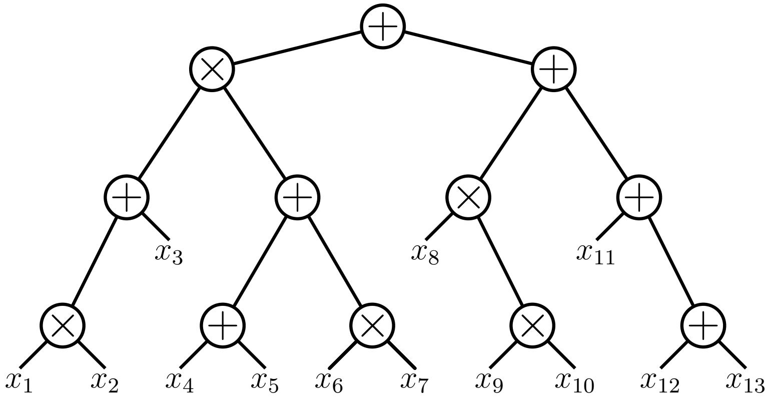

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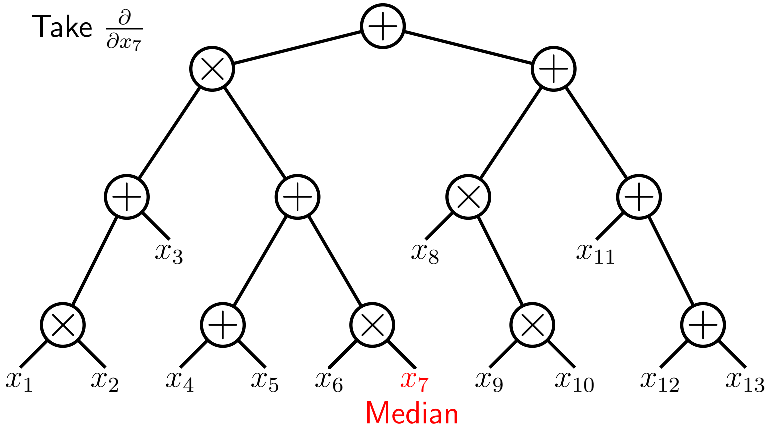

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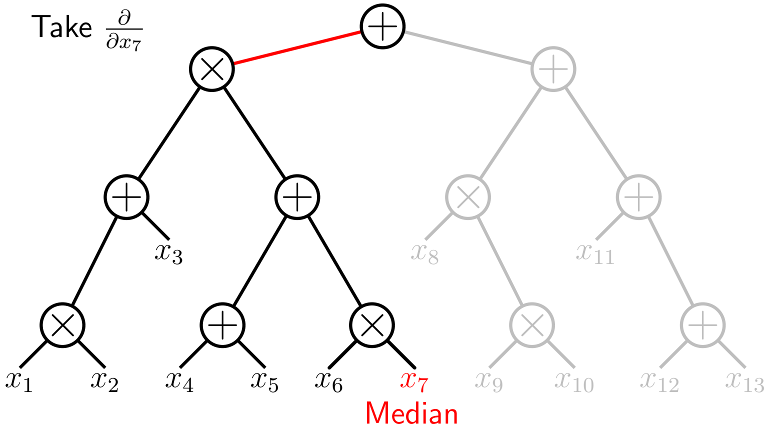

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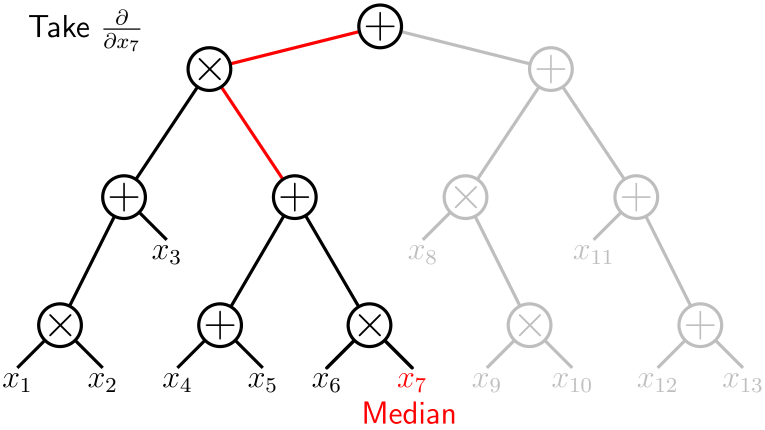

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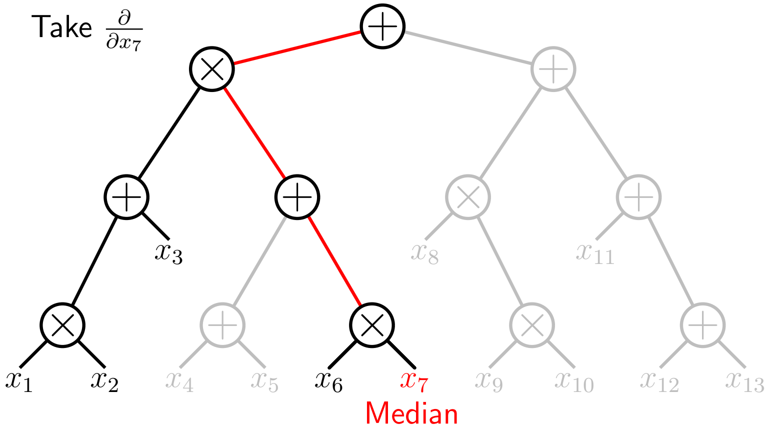

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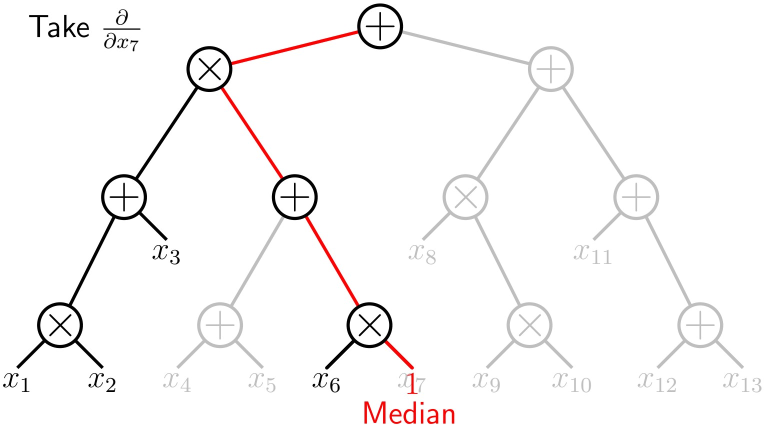

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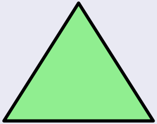

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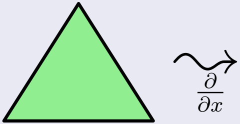

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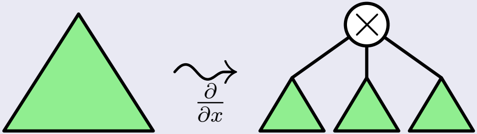

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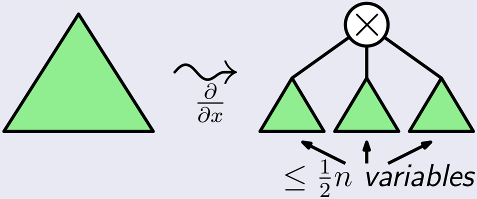

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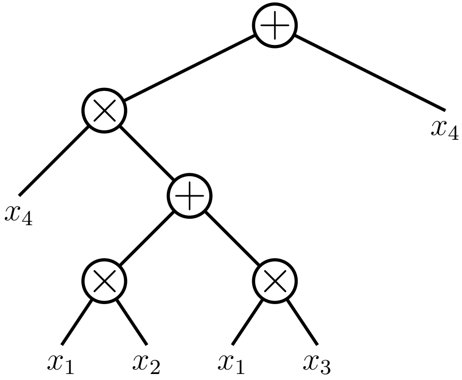

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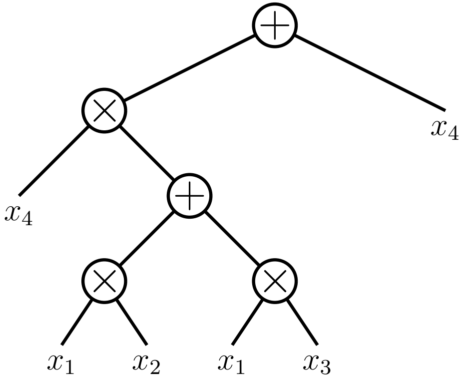

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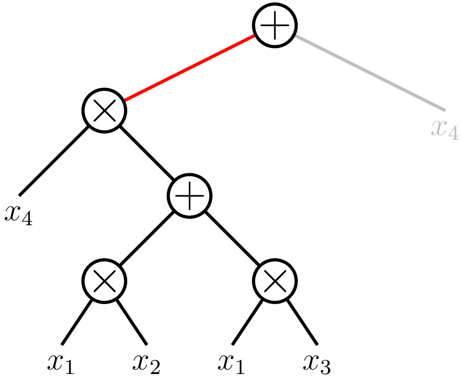

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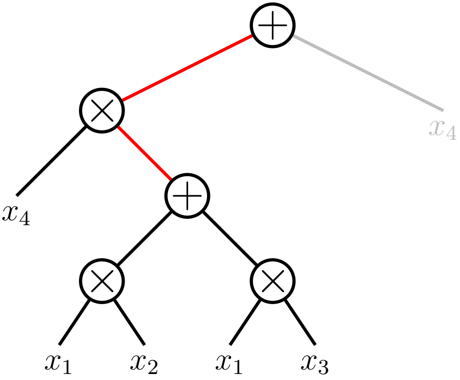

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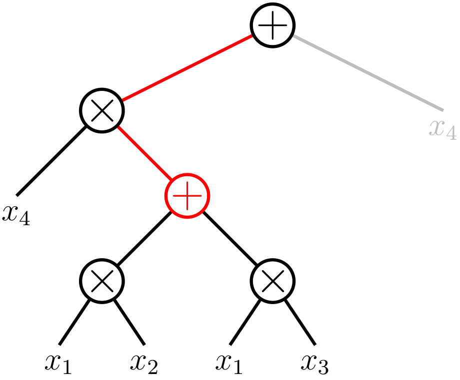

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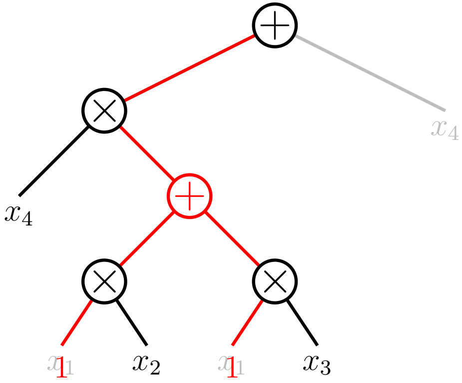

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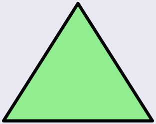

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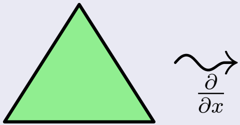

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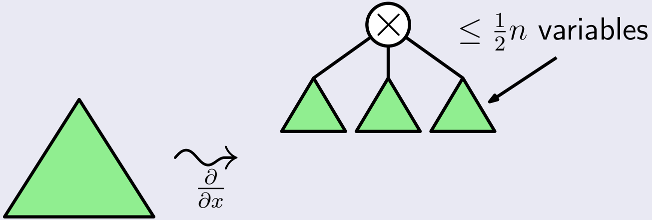

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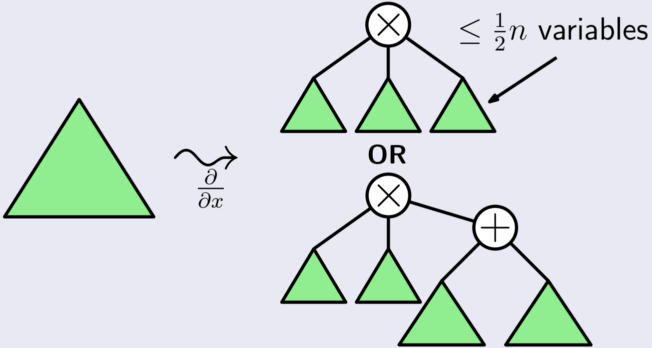

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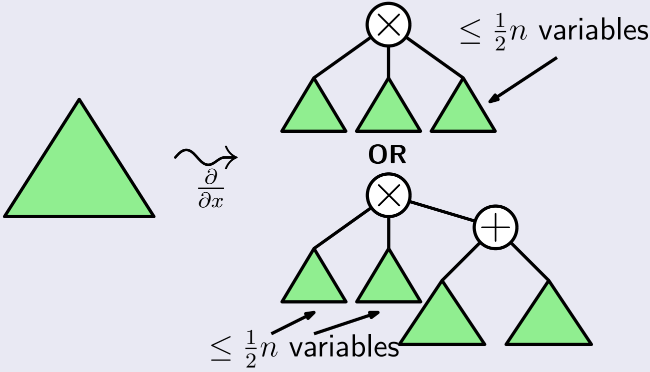

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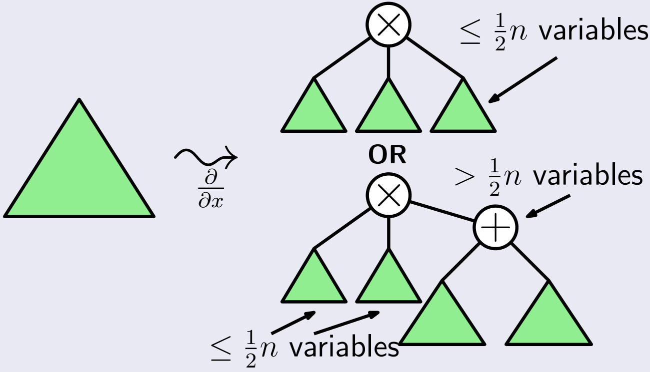

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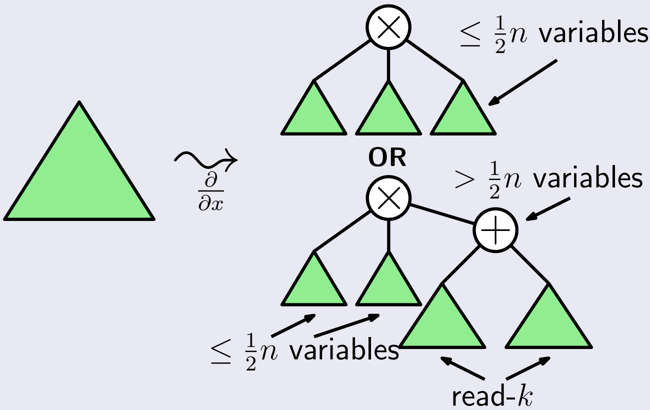

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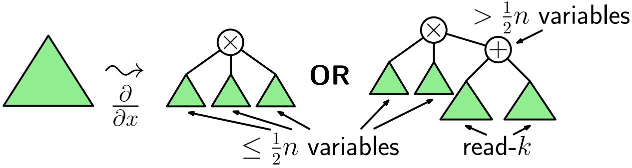

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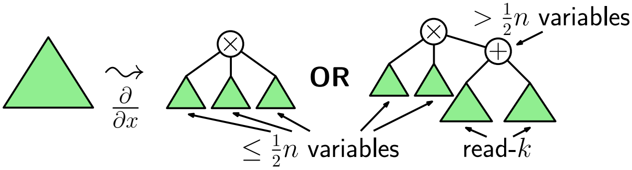

Testing $\sum^2$-read-$k \leq$ Testing read-k

Fact (SV Hitting Set [SV09])

The set of binary strings H_w with Hamming weight at most w hits any class $\mathcal{F}$ of multilinear polynomials that:

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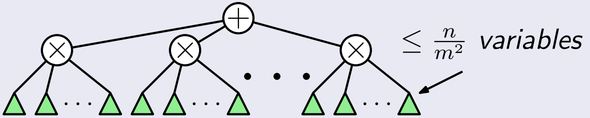

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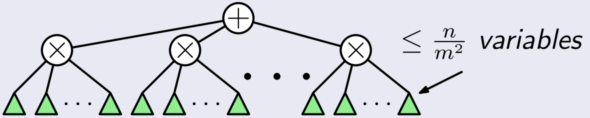
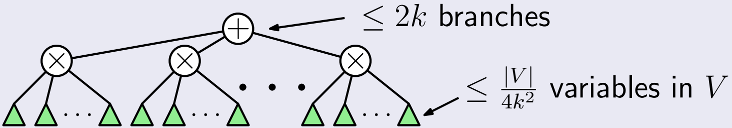


$\Rightarrow$ *F does not compute a monomial of degree n.*

The Shattering Lemma

Lemma (Shattering Lemma)

For any nonzero multilinear $\sum^2$-read-k formula F on n variables, there exist disjoint sets of variables P and V, with $|P| = \mathrm{poly}(k)$ and $|V| = \frac{n}{k^{O(k)}}$ such that $\frac{\partial F}{\partial P}$ is nonzero and can be written as



where each small subformula is the partial derivative of some subformula of F.

Theorem

1. *$F(\bar{x} + \bar{\sigma})$ is not a monomial.*
2. *$\bar{\sigma}$ is easy to compute.*

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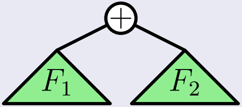

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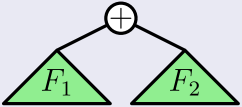
$\Rightarrow$ **Shatter(** 

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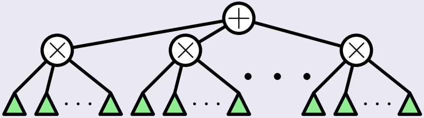
$\Rightarrow$  $\equiv M_{n'}$

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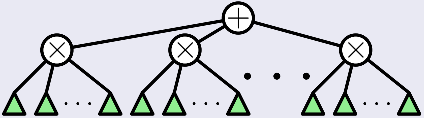
$\Rightarrow$  $\equiv M_{n'}$

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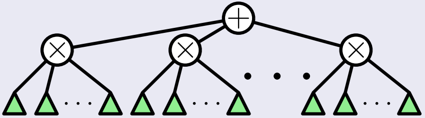
$\Rightarrow$  $\equiv M_{n'}$

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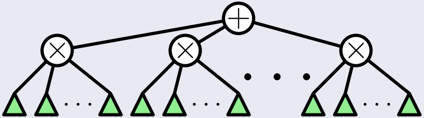
$\Rightarrow$  $\equiv M_{n'}$

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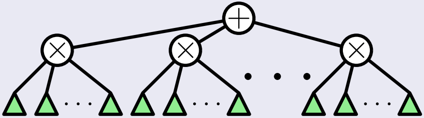
$\Rightarrow$  $\equiv M_{n'}$

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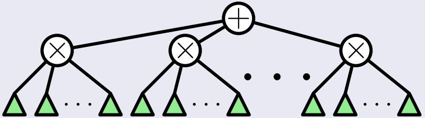
$\Rightarrow$  $\equiv M_{n'}$

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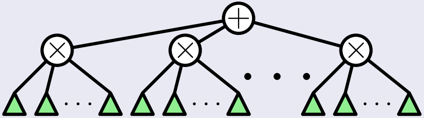
$\Rightarrow$  $\equiv M_{n'}$

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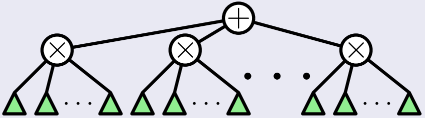
$\Rightarrow$  $\equiv M_{n'}$

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F is $\sum^2$-read-k, so $\bar{\sigma}$ can be computed efficiently using a read-k identity test. ∎

Outline

Techniques:

1. **Fragmenting**
   Reduces multilinear read-$(k+1)$ to multilinear $\sum^2$-read-k.
2. **Shattering**
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Theorem (Weakened Main)

There is a $s^{O(1)} \cdot n^{k^{O(k)}+O(k \log n)}$ time deterministic algorithm for identity testing n-variable size-s multilinear read-k formulae.

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Corollary

There is a polynomial-time deterministic algorithm for identity testing multilinear constant-read formulae.

Conclusion

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2. Sparse substituted: quasi-poly-time.
   - Encompasses depth-four multilinear formulae [KMSV10], and pre-processed $\sum^k$-read-once formulae [SV09].

Questions?

Thanks!

The full version of our paper may be found on ECCC.