In the undecidability proofs below, you may use the fact that $A_{TM}$, $HALT_{TM}$, and $E_{TM}$ are undecidable.

**Undecidability Proofs**

1. Show that the following language is undecidable:

   \[ \{ \langle M, a, w \rangle \mid M \text{ is a TM, } a \in \Gamma_M, \text{ and } w \in \Sigma_M^+ \text{ and when } M \text{ runs on } w \text{ it will at some point write } a \text{ onto the tape} \} \]

2. Show that the following language is undecidable:

   \[ A = \{ \langle M \rangle \mid M \text{ is a TM and there is some string that } M \text{ accepts} \} \]

3. Show that the following language is undecidable:

   \[ B = \{ \langle M \rangle \mid M \text{ is a TM and } M \text{ halts when started with a blank tape} \} \]

4. [Sipser [1], prob. 5.13, pg. 239] A **useless state** in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show that the language is undecidable.

**References**


**Honor Code Affirmation**

I affirm that I have carried out my academic endeavors with full academic honesty.