1. [From Sipser [1], p. 89, no. 1.35.] Let

\[ \Sigma_2 = \{0^r 1^s 1^t | r = s = t \}\]

\(\Sigma_2\) is all pairs of 0s and 1s, arranged in columns. A string on this alphabet can be viewed as two rows of 0s and 1s:

Consider the language:

\[ A = \{w \in \Sigma_2^* | \text{the bottom row of } w \text{ is the reverse of the top row of } w\} \]

Show that \(A\) is not regular.

2. [Sipser [1], p. 91, no. 1.53] Let \(\Sigma = \{0, 1, +, =\}\) and consider the following language:

\[ ADD = \{x = y + z | x, y, z \text{ are binary integers such that } x \text{ is the sum of } y \text{ and } z\} \]

Show that \(ADD\) is not regular.

3. [Sipser [1], p. 88, no. 1.30] Describe the error in the following “proof” that 0∗1∗ is not a regular language. (An error must exist because 0∗1∗ is regular.)

**Claim** 0∗1∗ is not regular.

**Proof.** Assume for contradiction that 0∗1∗ is regular. Let \(p\) be the pumping length for 0∗1∗ given by the pumping lemma. Let \(s = 0^p 1^p\). We know that \(s \in 0^* 1^*\) and that \(|s| \geq p\). Therefore, \(s\) can be pumped. However, we’ve previously shown that \(0^p 1^p\) cannot be pumped (see example 1.73, where we show that \(\{0^n 1^n | n \geq 0\}\) is not regular). We have reached a contradiction—\(s\) can be pumped and cannot be pumped. Therefore our original assumption is false. Therefore 0∗1∗ is not regular. \(\Box\)

4. [Sipser [1], p. 91, no. 1.54] Consider the language \(F = \{a^i b^j c^k | i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\} \).

(a) Show that \(F\) is not regular.

(b) Show that \(F\) acts like a regular language in the pumping lemma. In other words, give a pumping length \(p\) and demonstrate that \(F\) satisfies the three conditions of the pumping lemma for this value \(p\).

(c) Explain why parts 4a and 4b do not contradict the pumping lemma.

**REFERENCES**